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Democratic Universal Seesaw Model with Three Light Sterile Neutrinos

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Abstract

Based on the “democratic” universal seesaw model, where mass matrices M_f of quarks and leptons f_i ($f = u, d, \nu, e$; $i = 1, 2, 3$) are given by a seesaw form $M_f \simeq -m_L M_F^{-1} m_R$, and m_L and m_R are universal for all the fermion sectors, and the mass matrices M_F of hypothetical heavy fermions F_i have a democratic structure, a possible neutrino mass matrix is investigated. In the model, there are three sterile neutrinos ν_{iR} which mix with the active neutrinos ν_{iL} with $\theta \sim 10^{-2}$ and which are harmless for constraint from the big bang nucleosynthesis. The atmospheric, solar and the LSND neutrino data are explained by the mixings $\nu_{\mu L} \leftrightarrow \nu_{\tau L}$, $\nu_{eL} \leftrightarrow \nu_{eR}$ and $\nu_{eL} \leftrightarrow \nu_{\mu L}$, respectively. The model predicts that $\Delta m_{solar}^2 / \Delta m_{atm}^2 \simeq (R^2 - 1)m_e / \sqrt{m_\mu m_\tau}$ [$R = m(\nu_{iR}) / m(\nu_{iL})$ ($i = 1, 2, 3$)] with $\sin^2 2\theta_{atm} \simeq 1$ and $\Delta m_{LSND}^2 / \Delta m_{atm}^2 \simeq (1/4) \sqrt{m_\mu / m_e}$ with $\sin^2 2\theta_{LSND} \simeq 4m_e / m_\mu$.

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1 Introduction

In order to seek for a clue to the unified understanding of quarks and leptons, many attempts to give a unified description of the quark and lepton mass matrices have been proposed. The universal seesaw mass matrix model [1] is one of the promising attempts to view the unified description, where the mass matrices M_f for the conventional quarks and leptons f_i ($f = u, d, \nu, e$; $i = 1, 2, 3$) are given by

$$(\bar{f}_L \ \bar{F}_L) \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} \begin{pmatrix} f_R \\ F_R \end{pmatrix}, \quad (1.1)$$

and m_L and m_R are universal for all fermion sectors f . For $O(M_F) \gg O(m_R) \gg O(m_L)$, the mass matrix (1.1) leads to the well-known seesaw expression

$$M_f \simeq -m_L M_F^{-1} m_R. \quad (1.2)$$

As a specific version of such universal seesaw model, Fusaoka and one of the authors (Y.K.) have proposed a so-called “democratic” seesaw model [2]: The heavy fermion matrices M_F have a simple structure [(unit matrix)+(democratic matrix)], i.e.,

$$M_F = m_0 \lambda_f (\mathbf{1} + 3b_f X), \quad (1.3)$$

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (1.4)$$

on the basis on which the matrices m_L and m_R are diagonal:

$$m_L = \frac{1}{\kappa} m_R = m_0 Z = m_0 \begin{pmatrix} z_1 & 0 & 0 \\ 0 & z_2 & 0 \\ 0 & 0 & z_3 \end{pmatrix}, \quad (1.5)$$

where the parameters z_1 , z_2 and z_3 are normalized as $z_1^2 + z_2^2 + z_3^2 = 1$, and m_0 is of the order of the electroweak symmetry breaking scale, i.e., $m_0 \sim 10^2$ GeV. Since the parameter b_f in the charged lepton sector is taken as $b_e = 0$, the parameters z_i are fixed as

$$\frac{z_1}{\sqrt{m_e}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_\tau + m_\mu + m_e}}. \quad (1.6)$$

For the up-type quark sector, the parameter b_f is taken as $b_u = -1/3$, which leads to $\det M_U = 0$, and the seesaw mechanism does not work for one of the three families, and

hence we obtain the mass $m_t \simeq m_0/\sqrt{3}$ without the seesaw suppression factor κ/λ_u (we identify it as the top quark mass). Furthermore, we also obtain a relation $m_u/m_c \simeq 3m_e/m_\mu$, which is in good agreement with the observed values. Moreover, when we take $b_d \simeq -1$ ($b_d = -e^{i\beta_d}$ with $\beta_d = 18^\circ$) for the down-type quark sector, we can obtain reasonable quark mass ratios and the Cabbibo-Kobayashi-Maskawa [3] (CKM) matrix.

The neutrino mass matrix in the universal seesaw mass matrix model is given as follows:

$$\begin{pmatrix} \bar{\nu}_L & \bar{\nu}_R^c & \bar{N}_L & \bar{N}_R^c \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & m_L \\ 0 & 0 & m_R^T & 0 \\ 0 & m_R & M_{NL} & M_D \\ m_L^T & 0 & M_D^T & M_{NR} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \\ N_L^c \\ N_R \end{pmatrix}, \quad (1.7)$$

where $\psi_L^c \equiv (\psi_L)^c = C\psi_L^T$. [We consider a $\text{SO}(10)_L \times \text{SO}(10)_R$ model [4], where fermions $(f_L + F_R^c)$ and $(f_R + F_L^c)$ are assigned to $(16,1)$ and $(1,16)$ under $\text{SO}(10)_L \times \text{SO}(10)_R$, respectively. Hereafter, we will denote the Majorana mass matrices M_{NL} and M_{NR} of the neutral heavy leptons N_L and N_R as $M_R = M_{NL}$ and $M_L = M_{NR}$, respectively.]

For $O(m_L) \ll O(m_R) \ll O(M_D)$, $O(M_L)$, $O(M_R)$, we obtain the following 6×6 seesaw mass matrix for (ν_L^c, ν_R)

$$M^{(6 \times 6)} \simeq - \begin{pmatrix} 0 & m_L \\ m_R^T & 0 \end{pmatrix} \begin{pmatrix} M_R & M_D \\ M_D^T & M_L \end{pmatrix}^{-1} \begin{pmatrix} 0 & m_R \\ m_L^T & 0 \end{pmatrix}, \quad (1.8)$$

which leads to the 3×3 seesaw matrices for ν_L and ν_R

$$M(\nu_L) \simeq -m_L M_L^{-1} m_L^T, \quad (1.9)$$

$$M(\nu_R) \simeq -m_R M_R^{-1} m_R^T. \quad (1.10)$$

The scenario corresponding to $O(m_L M_L^{-1} m_L^T) \ll O(m_R M_R^{-1} m_R^T)$ has already been investigated by one of the authors (Y.K.) [5]. He has concluded that although either the atmospheric [6] or solar [7] neutrino data can be explained by the mixings $\nu_\mu \leftrightarrow \nu_\tau$ or $\nu_e \leftrightarrow \nu_\mu$, however, simultaneous explanation of the both data cannot be obtained in this model.

In the present paper, we consider another possibility $O(m_L M_L^{-1} m_L^T) \sim O(m_R M_R^{-1} m_R^T)$. In this case, mixings between ν_{iL} and ν_{iR} are induced. The solar neutrino data [7] are understood from a small mixing between ν_{eL} and ν_{eR} . The atmospheric [6] and the LSND [8] neutrino data are explained by the mixings $\nu_{\mu L} \leftrightarrow \nu_{\tau L}$ and $\nu_{eL} \leftrightarrow \nu_{\mu L}$, respectively. The vantage point of the democratic seesaw model [2] is that parameters z_i in the mass matrices m_L and m_R are given in terms of the charged lepton masses and thereby the

mass spectrum and mixings of ν_{iL} and ν_{iR} can also be predicted in terms of the charged lepton masses.

2 Parameter b_ν

In the present paper, for simplicity, we assume that all the neutral heavy fermion mass matrices M_D , M_L and M_R have the same flavor structure

$$\frac{1}{\lambda_D}M_D = \frac{1}{\lambda_L}M_L = \frac{1}{\lambda_R}M_R = m_0(\mathbf{1} + 3b_\nu X), \quad (2.1)$$

and we will investigate only the case $b_\nu = -1/2$.

The excuse for considering only the case $b_\nu = -1/2$ is as follows. The choices of b_f ($b_e = 0, b_u = -1/3, b_d \simeq -1$) have given the successful description of the quark masses and mixings in terms of the charged lepton masses. When, instead of the expression (1.3), we denote M_F as

$$M_F = m_0\lambda_f\sqrt{1 + 2b_f + 3b_f^2}(\cos\phi_f E - \sin\phi_f S), \quad (2.2)$$

$$E = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad S = \frac{1}{\sqrt{6}} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad (2.3)$$

where E and S have been normalized as $\text{Tr}E^2 = \text{Tr}S^2 = 1$ and $\tan\phi_f = -\sqrt{2}b_f/(1 + b_f)$, the cases $b_e = 0$, $b_u = -1/3$ and $b_d = -1$ correspond to $(\cos\phi_f, \sin\phi_f) = (1, 0)$, $(\sqrt{2/3}, \sqrt{1/3})$ and $(0, 1)$, respectively. Considering an empirical relation $\phi_d = \pi/2 - \phi_e$ for $(\cos\phi_e, \sin\phi_e) = (1, 0)$ and $(\cos\phi_d, \sin\phi_d) = (0, 1)$, we consider that the value of b_ν is also given by $\phi_\nu = \pi/2 - \phi_u$ for $(\cos\phi_u, \sin\phi_u) = (\sqrt{2/3}, \sqrt{1/3})$, i.e., we assume

$$(\cos\phi_\nu, \sin\phi_\nu) = (\sqrt{1/3}, \sqrt{2/3}), \quad (2.4)$$

which corresponds to the case $b_\nu = -1/2$.

Besides, from the phenomenological point of view, the case $b_\nu = -1/2$ is also interesting. The inverse matrix of the M_L with $b_\nu = -1/2$

$$M_L = m_0\lambda_L(\mathbf{1} - \frac{1}{2} \cdot 3X) = \frac{1}{2}m_0\lambda_L \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}, \quad (2.5)$$

is given by

$$M_L^{-1} = -\frac{1}{m_0\lambda_L} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad (2.6)$$

so that the seesaw matrix $M_\nu \simeq -m_L M_L^{-1} m_L^T$ is expressed as

$$M_\nu \simeq m_0 \frac{1}{\lambda_L} \begin{pmatrix} 0 & z_1 z_2 & z_1 z_3 \\ z_1 z_2 & 0 & z_2 z_3 \\ z_1 z_3 & z_2 z_3 & 0 \end{pmatrix}. \quad (2.7)$$

The form (2.7) is just a Zee-type mass matrix [9], which has recently been revived [10] as a promising neutrino mass matrix form.

3 Mass spectrum and mixing

For the specific form (2.1) with $b_\nu = -1/2$, the 6×6 seesaw matrix $M^{(6 \times 6)}$ given by Eq. (1.8) becomes

$$\begin{aligned} M^{(6 \times 6)} &\simeq -m_0 \begin{pmatrix} 0 & Z \\ \kappa Z & 0 \end{pmatrix} \begin{pmatrix} \lambda_R Y & \lambda_D Y \\ \lambda_D Y & \lambda_L Y \end{pmatrix}^{-1} \begin{pmatrix} 0 & \kappa Z \\ Z & 0 \end{pmatrix} \\ &= -m_0 \frac{1}{\lambda_R \lambda_L - \lambda_D^2} \begin{pmatrix} \lambda_R Z Y^{-1} Z & -\kappa \lambda_D Z Y^{-1} Z \\ -\kappa \lambda_D Z Y^{-1} Z & \kappa^2 \lambda_L Z Y^{-1} Z \end{pmatrix}, \end{aligned} \quad (3.1)$$

where

$$Y = \mathbf{1} + 3b_\nu X, \quad Y^{-1} = \mathbf{1} + 3a_\nu X, \quad (3.2)$$

$$a_\nu = -b_\nu / (1 + 3b_\nu). \quad (3.3)$$

Therefore, the matrix $M^{(6 \times 6)}$ is diagonalized by the 6×6 unitary matrix $U^{(6 \times 6)}$

$$U^{(6 \times 6)} = \begin{pmatrix} \cos \theta \cdot U & -\sin \theta \cdot U \\ \sin \theta \cdot U & \cos \theta \cdot U \end{pmatrix}, \quad (3.4)$$

as

$$\begin{aligned} U^{(6 \times 6) \dagger} M^{(6 \times 6)} U^{(6 \times 6)} &= \text{diag}(m_{\nu_{1L}}, m_{\nu_{2L}}, m_{\nu_{3L}}, m_{\nu_{1R}}, m_{\nu_{2R}}, m_{\nu_{3R}}) \\ &= m_0 \text{diag}(\xi_L \rho_1, \xi_L \rho_2, \xi_L \rho_3, \xi_R \rho_1, \xi_R \rho_2, \xi_R \rho_3), \end{aligned} \quad (3.5)$$

where

$$U^\dagger ZY^{-1}ZU = \text{diag}(\rho_1, \rho_2, \rho_3), \quad (3.6)$$

$$\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \lambda_R & -\kappa \lambda_D \\ -\kappa \lambda_D & \kappa^2 \lambda_L \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} = \begin{pmatrix} \lambda'_L & 0 \\ 0 & \lambda'_R \end{pmatrix}, \quad (3.7)$$

$$\xi_L = \frac{\lambda'_L}{\lambda_R \lambda_L - \lambda_D^2}, \quad \xi_R = \frac{\lambda'_R}{\lambda_R \lambda_L - \lambda_D^2}, \quad (3.8)$$

$$\begin{pmatrix} \lambda'_L \\ \lambda'_R \end{pmatrix} = \frac{1}{2}(\lambda_R + \kappa^2 \lambda_L) \mp \frac{1}{2}(\lambda_R - \kappa^2 \lambda_L) \sqrt{1 + \tan^2 2\theta}. \quad (3.9)$$

The mixing angle θ between ν_{iL} and ν_{iR} is given by

$$\tan 2\theta = \frac{2\kappa \lambda_D}{\lambda_R - \kappa^2 \lambda_L}. \quad (3.10)$$

The light neutrino masses $m(\nu_{iL})$ and $m(\nu_{iR})$ are given by

$$m(\nu_{iL}) = m_0 \xi_L \rho_i, \quad m(\nu_{iR}) = m_0 \xi_R \rho_i. \quad (3.11)$$

For the case of $b_\nu = -1/2$, the eigenvalues ρ_i of the matrix $ZY^{-1}Z$ are given by

$$\rho_1 \simeq -2z_1^2, \quad \rho_2 \simeq -\left(z_2 + \frac{z_1^2}{2z_2} - z_1^2\right), \quad \rho_3 \simeq z_2 + \frac{z_1^2}{2z_2} + z_1^2, \quad (3.12)$$

so that

$$\rho_3^2 - \rho_2^2 \simeq 4z_2 z_1^2, \quad \rho_2^2 - \rho_1^2 \simeq z_2^2. \quad (3.13)$$

The 3×3 mixing matrix U for the case $b_\nu = -1/2$ is given by

$$U \simeq \begin{pmatrix} -1 & -\frac{1}{\sqrt{2}} \frac{z_1}{z_2} (1 - z_2) & \frac{1}{\sqrt{2}} \frac{z_1}{z_2} (1 + z_2) \\ \frac{z_1}{z_2} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ z_1 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (3.14)$$

4 Explanations of the neutrino data

The atmospheric [6] and solar [7] neutrino data are explained by the mixings $\nu_{\mu L} \leftrightarrow \nu_{\tau L}$ and $\nu_{eL} \leftrightarrow \nu_{eR}$, respectively. As seen in the mixing matrix (3.14), the neutrinos $\nu_{\mu L}$

and $\nu_{\tau L}$ are maximally mixed. On the other hand, the mixing between ν_{eL} and ν_{eR} is given by Eq. (3.10). Since the solar neutrino data disfavor [11] sterile neutrino with a large mixing angle, we take the small mixing angle solution in the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism [12],

$$\Delta m_{solar}^2 \simeq 4.0 \times 10^{-6} \text{ eV}^2, \quad \sin^2 2\theta_{solar} \simeq 6.9 \times 10^{-3}. \quad (4.1)$$

Here, the values in Eq. (4.1) have been quoted from the recent analysis for $\nu_e \rightarrow \nu_s$ by Bahcall, Krastev and Smirnov [13]. The value $\sin^2 2\theta_{solar} \simeq 7 \times 10^{-3}$ can be fitted by adjusting the parameters λ_L , λ_R/κ^2 and λ_D/κ in Eq. (3.10).

As seen from Eqs. (3.5) and (3.13), the ratio of $\Delta m_{solar}^2 = (m_{\nu_{1R}})^2 - (m_{\nu_{1L}})^2$ to $\Delta m_{atm}^2 = (m_{\nu_{3L}})^2 - (m_{\nu_{2L}})^2$ is given by

$$\frac{\Delta m_{solar}^2}{\Delta m_{atm}^2} \simeq \frac{\lambda_R'^2 - \lambda_L'^2}{\lambda_L'^2} \frac{4z_1^4}{4z_2 z_1^2} \simeq (R^2 - 1) \frac{m_e}{\sqrt{m_\mu m_\tau}} = (R^2 - 1) \times 1.15 \times 10^{-3}, \quad (4.2)$$

where

$$R = \frac{\lambda_R'}{\lambda_L'} = \frac{\xi_R}{\xi_L} = \frac{m(\nu_{iR})}{m(\nu_{iL})}. \quad (4.3)$$

The recent best fit value $\Delta m_{atm}^2 = 3.2 \times 10^{-3} \text{ eV}^2$ [14] gives the ratio

$$\frac{\Delta m_{solar}^2}{\Delta m_{atm}^2} \simeq \frac{4.0 \times 10^{-6} \text{ eV}^2}{3.2 \times 10^{-3} \text{ eV}^2} \simeq 1.3 \times 10^{-3}. \quad (4.4)$$

By comparing Eqs. (4.2) and (4.4), we obtain $R \simeq 1.4$. Note that the observed value (4.4) is in good agreement with the value $m_e/\sqrt{m_\mu m_\tau}$, so that we are tempted to consider a model with $R \simeq 0$. However, the sign of Δm_{solar}^2 in the small mixing angle MSW solution must be positive, so that we cannot consider the case $R \simeq 0$. In the present model, R is only a phenomenological parameter with the constraint $R > 1$.

The LSND data [8] is explained by the mixing $\nu_{eL} \leftrightarrow \nu_{eR}$. The mass-squared difference $\Delta m_{LSND}^2 = m_{\nu_{2L}}^2 - m_{\nu_{1L}}^2$ and the $\nu_{eL} \leftrightarrow \nu_{eR}$ mixing angle are given by the ratio

$$\frac{\Delta m_{LSND}^2}{\Delta m_{atm}^2} \simeq \frac{z_2}{4z_1} \simeq \frac{1}{4} \sqrt{\frac{m_\mu}{m_e}} = 2.2 \times 10^2, \quad (4.5)$$

and

$$\sin^2 2\theta_{LSND} \simeq 4U_{e1}^2 U_{\mu 1}^2 \simeq 4 \left(\frac{z_1}{z_2} \right)^2 \simeq 4 \frac{m_e}{m_\mu} \simeq 0.019, \quad (4.6)$$

respectively. The best fit value $\Delta m_{atm}^2 \simeq 3.2 \times 10^{-3} \text{ eV}^2$ give a prediction $\Delta m_{LSND}^2 \simeq 0.70 \text{ eV}^2$. However, the region $\Delta m_{LSND}^2 \geq 0.34 \text{ eV}^2$ in the LSND favored region at $\sin^2 2\theta = 0.02$ has been excluded by the recent KARMEN2 experiment [15]. Therefore, only when we take the value $\Delta m_{LSND}^2 \simeq 0.33 \text{ eV}^2$, we can obtain the prediction $\Delta m_{atm}^2 \simeq 1.5 \times 10^{-3} \text{ eV}^2$ which is barely inside the 90% C.L. allowed region ($1.5 \times 10^{-3} \text{ eV}^2 \leq \Delta m_{atm}^2 \leq 5 \times 10^{-3} \text{ eV}^2$) in the recent Super-Kamiokande atmospheric neutrino data [14]. Hereafter, we will adopt this pinpoint solution:

$$\Delta m_{LSND}^2 \simeq 0.33 \text{ eV}^2, \quad \Delta m_{atm}^2 \simeq 1.5 \times 10^{-3} \text{ eV}^2. \quad (4.7)$$

Then, the parameter R is fixed as

$$R \simeq 1.8, \quad (4.8)$$

from Eq. (4.2), and the neutrino masses are predicted as follows:

$$m(\nu_{3L}) \simeq m(\nu_{2L}) \simeq 0.57 \text{ eV}, \quad m(\nu_{1L}) \simeq 1.3 \times 10^{-3} \text{ eV}, \quad (4.9)$$

$$m(\nu_{3R}) \simeq m(\nu_{2R}) \simeq 1.05 \text{ eV}, \quad m(\nu_{1R}) \simeq 2.5 \times 10^{-3} \text{ eV}, \quad (4.10)$$

where we have used the relation $m(\nu_{2L}) \simeq \sqrt{\Delta m_{LSND}^2}$.

In the present scenario, there are three light sterile neutrinos ν_{iR} ($i = 1, 2, 3$). However, those neutrinos do not spoil the big bang nucleosynthesis (BBN) scenario, which puts the following constraint [16] for a mixing between the active neutrino ν_α ($\alpha = e, \mu, \tau$) and a sterile neutrino ν_s ,

$$(\sin^2 2\theta_{\alpha s})^2 \Delta m_{\alpha s}^2 < 3.6 \times 10^{-4} \text{ eV}^2. \quad (4.11)$$

The value of $(\sin^2 2\theta)^2 \Delta m^2$ in our model is less than 10^{-4} eV^2 , because the mixing angle θ in the present model is sufficiently small, i.e., $(\sin^2 2\theta)^2 = (6.9 \times 10^{-3})^2 = 4.8 \times 10^{-5}$.

However, we have another severe constraint on the neutrino masses from the cosmic structure formation in a low-matter-density universe [17]

$$N_\nu m_\nu < 1.8 \text{ eV} \quad (1.5 \text{ eV}), \quad (4.12)$$

for flat universe (for open universes), where N_ν is the number of almost degenerate neutrinos with the highest mass. The present model gives $N_\nu m_\nu \simeq 3.2 \text{ eV}$, so that the model dose not satisfies the constraint (4.12). We will go optimistically for this problem.

The mixing between ν_{eL} and $\nu_{\tau L}$ is given by

$$U_{e3} \simeq \frac{1}{\sqrt{2}} \frac{z_1}{z_2} (1 + z_2) \simeq \sqrt{\frac{m_e}{2m_\mu}} \left(1 + \sqrt{\frac{m_\mu}{m_\tau}} \right) \simeq 0.061, \quad (4.13)$$

which safely satisfies the constraint $|U_{e3}| \leq (0.22 - 0.14)$ obtained from the CHOOZ reactor neutrino experiment [18].

5 Conclusion and discussion

In conclusion, we have investigated a neutrino mass matrix in the framework of the “democratic” universal seesaw model. Although the model has three light sterile neutrinos ν_{iR} ($i = 1, 2, 3$), they do not spoil the BBN scenario, because the mixing angle θ between the active and sterile neutrinos is taken as $\sin^2 2\theta \simeq 7 \times 10^{-3}$. The atmospheric, solar and LSND neutrino data are explained by the mixings $\nu_{\mu L} \leftrightarrow \nu_{\tau L}$, $\nu_{eL} \leftrightarrow \nu_{eR}$ and $\nu_{eL} \leftrightarrow \nu_{\mu L}$, respectively. The model with the parameter $b_\nu = -1/2$ gives the predictions in terms of the charged lepton masses,

$$\frac{\Delta m_{solar}^2}{\Delta m_{atm}^2} \simeq (R^2 - 1) \frac{m_e}{\sqrt{m_\mu m_\tau}} , \quad \frac{\Delta m_{LSND}^2}{\Delta m_{atm}^2} \simeq \frac{1}{4} \sqrt{\frac{m_\mu}{m_e}} , \quad (5.1)$$

$$\sin^2 2\theta_{atm} \simeq 1 , \quad \sin^2 2\theta_{LSND} \simeq 4 \frac{m_e}{m_\mu} , \quad (5.2)$$

where $R = m(\nu_{iR})/m(\nu_{iL})$. In the present model, the prediction $\Delta m_{solar}^2/\Delta m_{atm}^2$ includes a free parameter R . Only a parameter independent prediction is $\Delta m_{LSND}^2/\Delta m_{atm}^2$ together with $\sin^2 2\theta_{LSND} \simeq 4m_e/m_\mu$. Since the most part of the allowed region of the ν_e - ν_μ oscillation in the LSND data is ruled out by the KARMEN2 data [15] (but a narrow region still remains), the predictability of the present model is somewhat faded from the point of view of the neutrino phenomenology. However, the motivation of the present paper is not to give the explanation of the LSND data, but to seek for a possible unification model of the quark and lepton mass matrices. The presence of the light right-handed neutrinos ν_{iR} will offer fruitful new physics to the near future neutrino experiments.

In the present scenario, the following intermediate energy scales have been considered: The neutral leptons N_L and N_R acquire large Majorana masses M_R and M_L at $\mu = \Lambda_{NL} = m_0\lambda_R$ and $\mu = \Lambda_{NR} = m_0\lambda_L$, respectively. The fermions N and F ($F = U, D, E$) acquire large Dirac masses M_D and M_F at $\mu = \Lambda_D = m_0\lambda_D$ and $\mu = \Lambda_F = m_0\lambda_F$, respectively. The gauge symmetries $SU(2)_R$ and $SU(2)_L$ are broken at $\mu = \Lambda_R = m_0\kappa$ and $\mu = \Lambda_L = m_0$, respectively. For $\tan^2 2\theta \ll 1$, from Eq. (3.9), we obtain the approximate relations

$$\lambda'_L \simeq \kappa^2 \lambda_L , \quad \lambda'_R \simeq \lambda_R , \quad (5.3)$$

so that

$$R = \frac{\lambda'_R}{\lambda'_L} \simeq \frac{\lambda_R}{\kappa^2 \lambda_L} . \quad (5.4)$$

The numerical result $R = O(1)$ means $\lambda_R/\lambda_L \sim \kappa^2$, i.e.,

$$\frac{\Lambda_{NL}}{\Lambda_{NR}} \sim \left(\frac{\Lambda_R}{\Lambda_L}\right)^2. \quad (5.5)$$

Since

$$m(\nu_{2L}) = \xi_L \rho_2 m_0 \simeq \frac{\kappa^2 \lambda_L}{\lambda_R \lambda_L - \lambda_D^2} \rho_2 m_0 \simeq \frac{1}{\lambda_R / \kappa^2} \sqrt{\frac{m_\mu}{m_\tau}} m_0, \quad (5.6)$$

we estimate

$$\frac{\lambda_R}{\kappa^2} \simeq \sqrt{\frac{m_\mu}{m_\tau}} \frac{m_0}{m(\nu_{2L})} \sim 10^{11}, \quad (5.7)$$

where we have used $m_0 \sim 10^2$ GeV, so that we obtain $\Lambda_{NL} \sim \kappa^2 \times 10^{13}$ GeV. If we consider that Λ_{NL} must be smaller than the Planck mass $M_P \sim 10^{19}$ GeV, we obtain the constraint

$$\kappa \equiv \Lambda_R / \lambda_L < 10^3. \quad (5.8)$$

Since the case $\kappa \sim 1$ is experimentally ruled out, we conclude that

$$O(10^3) \text{ GeV} < \Lambda_R < O(10^5) \text{ GeV}. \quad (5.9)$$

From (3.10), we estimate

$$\frac{\lambda_D}{\kappa} \simeq \frac{1}{2} \left(1 - \frac{1}{R}\right) \frac{\lambda_R}{\kappa^2} \tan 2\theta \sim 10^9. \quad (5.10)$$

On the other hand, we have known that

$$\frac{\Lambda_R}{\Lambda_F} = \frac{\kappa}{\lambda_F} \sim 10^{-2} \quad (5.11)$$

from the study of the quark mass spectrum [2]. Therefore, we cannot take an idea that the Dirac masses M_D and M_F ($F \neq N$) are generated at the same energy scale $\mu = \Lambda_D = \Lambda_F$.

Note that in the conventional universal seesaw model, the neutrino masses are of the order of $\Lambda_L^2 / \Lambda_{NR} = m_0 / \lambda_L$, because of $M(\nu_L) \simeq m_L M_L^{-1} m_L^T$, so that we consider $\lambda_L \sim 10^9$. In contrast with the conventional model, in the present model, the value of λ_L is $\lambda_L \sim \lambda_R / \kappa^2 \sim 10^{11}$. Therefore, for example, the conclusion on the intermediate energy scales based on the $SO(10)_L \times SO(10)_R$ model in Ref. [19] is not applicable to the present model, because in Ref. [19] the solutions have been investigated under the condition $\lambda_L \sim 10^9$. It is a future task to seek for a unification model which satisfies these constraints on the intermediate energy scales, (5.5) and (5.7)-(5.11).

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